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GREENLAND'S RAPID POSTGLACIAL EMERGENCE: A RESULT OF ICE-WATER --ETC(U)  
AUG 76 C TAPSCOTT

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Greenland's rapid postglacial emergence:  
A result of ice-water gravitational attraction:  
Comment and reply

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↳ This note comments upon a paper published in 'Geology', v.4 no.5, 1976, in which J. A. Clark presented

Clark (1976) presents a simple method by which to calculate the change in sea level caused by the gravitational attraction of an ice cap. In the same paper he also presents some results of a more complex numerical solution of the problem, and the two calculations are in disagreement. <sup>(Tapscott points out that,</sup> In fact, his <sup>Clark's</sup> more powerful solution is correct. His simple calculation contains an error. <sup>The present note points out</sup> It may be useful to point out the reason for <sup>the error</sup> this, lest doubt be cast on his numerical method and the many interesting consequences of his thesis. ✓

When a point mass M is created on the surface of a perfectly rigid earth, Clark concludes that the change in sea level  $\theta$  degrees away from the mass is

$$S(\theta) = \frac{GM}{2ag} \csc\left(\frac{\theta}{2}\right)$$

where G is the gravitational constant, a is the radius of the earth, and g is the acceleration of gravity at the earth's surface. This might better be written as

$$S(\theta) = \frac{ax}{2} \csc\left(\frac{\theta}{2}\right)$$

where x is the ratio of M to the mass of the earth. This equation is obviously incorrect, as s is always positive, leading to a rise in sea level everywhere. This cannot happen if the

mass of the ocean is conserved, as Clark demands. The error enters Clark's derivation, described in his Appendix 1, in the equation  $\varphi^*(s) = \varphi_0$ ; that is, that the gravitational potential on the sea surface equipotential after the creation of the mass is equal to that on the sea surface equipotential before the creations of the mass. If the mass of the ocean is to be held constant this will not be true. The two sea surfaces will, of course, be equipotential surfaces, but their potential values (relative to a zero value at infinite radius) will in general differ.

Clark's equation does give the correct shape for an equipotential surface near the sea level surface, and one can use it to reach the correct solution by making two assumptions:

1) In the region of sea level, the gravitational gradient is of equal magnitude everywhere.

2) In terms of the variable  $\theta$ , oceans and continents are uniformly distributed over the earth.

The first of these assumptions has already been made by neglecting the partial derivatives of  $\varphi^*$  in Clark's equation A3 and by equating  $\frac{\partial \varphi_0}{\partial r}$  with  $-g$ . One can then say

$$S(\theta) = \frac{ax}{2} \csc\left(\frac{\theta}{2}\right) + C$$



where  $C$  is a constant whose value can be determined by requiring no net change in the volume of the ocean,

$$\int_0^\pi S(\theta) \sin \theta \, d\theta = 0.$$

This leads to

$$S(\theta) = \frac{ax}{2} \left\{ \csc \left( \frac{\theta}{2} \right)^{-2} \right\}$$

Thus, gravitational forces cause sea level to rise over the area within  $60^\circ$  of the mass and to fall over the rest of the sphere. One can get an idea of the significance of the second assumption by noting that over a wide range of ocean distributions (from the near hemisphere all ocean, and the far hemisphere all continent to the opposite case),  $C$  varies from  $-2.83 \frac{ax}{2}$  to  $-1.17 \frac{ax}{2}$ .

Clark uses his method to calculate the gravitation component of sea level change near Greenland due to the melting of Greenland ice cap since 9000 years B.P. He finds a 27 m component. If one uses the more correct equation, one finds a 25 m component. The difference between the two results is small because the mass of ice involved is small ( $\frac{ax}{2} = 1.07 \text{ m}$ ). A much larger mass of ice, however, can lead to serious discrepancies. Using a more complex numerical method, Clark considers the melting of the Laurentide and Fennoscandian ice caps. This nu-

merical approach includes the effects of the lateral extent of the ice caps and the self-gravitation of the oceans. If the melting raises the average sea level by 85 m ( $\frac{ax}{2} = 16.4m$ ), Clark's more simple method predicts a sea level rise of 69 m in the South Pacific and 52 m about 60° from the ice cap center. This disagrees with the results of his numerical analysis, as shown in his Figure 2. The method derived here, however, predicts a rise of 101 m in the South Pacific and 85 m about 60° from the ice cap center, in complete agreement with Clark's numerical solution. This indicates that Clark's numerical approach is correct, in spite of the error in his simple calculation. If Clark has not included the land-sea distribution in his solution, however, and he does not state that he has, he should do so before continuing with his stated plan of refining his model to look at small fluctuations in ice sheets. While the correction will have little effect on the size of small fluctuations in local sea level, it will noticeably effect large changes. Including the correction will be generally useful and will add to the value of the result.

#### REFERENCES CITED

- Clark, J. A., 1976, Greenland's rapid postglacial emergence: A result of ice-water gravitational attraction: *Geology*, v. 4, no. 5, p. 310-312.